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PROBLEM OF HEAT TRANSFER WITH ALLOWANCE FOR AXIAL HEAT CONDUCTION FOR THE FLOW OF A LIQUID IN TUBES AND CHANNELS

P. V. Tsoi UDC 536.24

The author presents a computational method for problems of heat transfer in channels with simple and complex two-dimensional profiles of the cross sections for the flow of liquid metals where axial heat conduction is taken into account. A comparison with solutions of the heat-transfer equation without the term that takes into account the conduction along the flowing liquid is given.

The combined heat transfer equation for the steady-state regime of flow of a medium inside a plane-parallel channel ($m = 0$, $-R \le x \le R$) and a round tube ($m = 1$, $0 \le r \le R$) in the relative coordinates ξ ($-1 \le \xi = x/R \le 1$, $0 \le \xi = r/R \le 1$) and $X = \frac{1}{\text{Pe}}\frac{z}{R}$, $\text{Pe} = w_0R/a$ and the time $\text{Fo} = at/R^2$ under symmetric conditions of thermal loadings is reduced to the form

$$\frac{\partial T}{\partial \text{Fo}} + w(\xi, m)\frac{\partial T}{\partial X} = \frac{1}{\xi^m}\frac{\partial}{\partial \xi}\left(\xi^m \frac{\partial T}{\partial \xi}\right) + \frac{1}{\text{Pe}^2}\frac{\partial^2 T}{\partial X^2} + \frac{q_v(\xi)\, R^2}{\lambda} f(X, \text{Fo}) . \qquad (1)$$

In the formulation of classical Grätz–Nusselt boundary-value problems the change in the heat flux due to the heat conduction along the flow of the liquid is considered to be small compared to the convective transfer of heat in this direction, i.e., the assumption

$$w(\xi, m)\frac{\partial T}{\partial X} >> \frac{1}{\text{Pe}^2}\frac{\partial^2 T}{\partial X^2}$$

is adopted, and Eq. (1) is solved without the second term in the right-hand side. Since $\text{Pe} = \frac{w_0 R}{a} = \frac{w_0 R}{\nu}\frac{\nu}{a} =$ Re Pr, for gases ($\text{Pr} \approx 1$) and nonmetallic liquids ($1 < \text{Pr} < 1000$) where $\text{Pe} > 25$ [1] this condition is satisfied almost without exception. However for liquid metals ($0.005 < \text{Pr} < 0.05$) with small Pe numbers the term $\delta \frac{\partial^2 T}{\partial X^2}$, $\delta = \text{Pe}^{-2}$ cannot be disregarded.

As $\delta \to \infty$ Eq. (1) in the variable X ($0 \le X \le \infty$) passes into the parabolic class, and for the stationary regime ($\partial T/\partial \text{Fo} = 0$) simultaneous use of Laplace integral transformation of the parabolic variable X, which is one-sided in terms of [2], and orthogonal projection of the discrepancy along the two-sided elliptic coordinate ξ (along ξ and η for channels with a two-dimensional cross-sectional profile) leads to synthesis of the transfer function of the sought temperature field by sums of blocks of elementary thermal-inertial links, while realization of this solving algorithm using double Laplace–Carson transformation of the variables X and Fo

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leads to synthesis of the solution in the form of an expansion in summands of responses of thermal-inertial links with the two-parameter transfer functions

$$\bar{u}_k^* (s, p) = \frac{A_k \bar{\Phi}^* (s, p)}{p + \gamma_k^{(n)} s + p_k^{(n)}} , \quad k = 1, 2, ..., n ,$$

to the thermal loadings $\Phi(X, \text{Fo})$. Here $\gamma_k^{(n)} = p_k^{(n)}/s_k^{(n)}$, $p_k^{(n)}$ and $s_k^{(n)}$ are the eigenvalues of the problems of nonstationary heat conduction inside a prismatic body and stationary heat transfer in a channel with the same profiles of the cross sections D, for example, for Eq. (1) – inside a round bar and a round tube.

Generalizations of the solving algorithm [3] to solutions of the telegraph equation of heat conduction

$$\delta_1 \frac{\partial T}{\partial \text{Fo}} + \delta_2 \frac{\partial^2 T}{\partial \text{Fo}^2} = \frac{\partial^2 T}{\partial \xi^2} + \beta \frac{\partial^2 T}{\partial \eta^2} + \frac{q_v (\xi, \eta)\, h^2}{\lambda} f (\text{Fo}) , \tag{2}$$

where δ_1 and δ_2 are correction parameters ($\delta_1 = 1$ and $\delta_2 = \text{Fo}_\tau = a\tau/h^2$, τ is the relaxation time [4]), led, in the domain of Laplace transforms, to synthesis of the temperature fields by sums of terms of elementary inertial-vibrational links ($\delta_1 = 1$ and $\delta_2 \neq 0$) or pure vibrational links (wave, $\delta_1 = 0$ and $\delta_2 \neq 0$) determined, respectively, by the expressions

$$u_k^* (p) = \frac{A_k \Phi^* (p)}{\delta_2 p^2 + \delta_1 p + z_k^{(n)}} , \quad u_k^* (p) = \frac{B_k \Phi^* (p)}{\delta_2 p^2 + z_k^{(n)}} , \tag{3}$$

where $z_1^{(n)} > 0$, $z_2^{(n)} > 0$, ..., $z_n^{(n)} > 0$ are approximate eigenvalues of the classical heat-conduction equation (2) ($\delta_1 = 1$, $\delta_2 = 0$) that coincide with the exact ones in the subscripts up to $k \leq n - 2$.

Equations (1) for $\delta = 0$ and (2) for $\delta_2 = 0$ and $\delta_1 \neq 0$ are included in the parabolic class and provide the basis for composing mathematical models for inertial systems of continuum mechanics inside which changes follow the fundamental laws of the thermodynamics of irreversible processes. In such equations, partial derivatives with respect to one-sided parabolic variables are an order lower than in hyperbolic equations. Therefore it seems impossible, from the solution of the complete equation (2), to find the solution for the classical heat-conduction equation by passing to the limit as $\delta_2 \to 0$. For the same reason we cannot obtain the limiting solution in singular degeneracy of the coefficient of the highest derivative $\partial^2 T/\partial X^2$ ($\text{Pe}^{-2} = \delta \to 0$) from the solution of Eq. (1), which is included in the class of equations of elliptic type.

Let us determine the solution of Eq. (1) under the stationary regime, i.e., $\partial T/\partial \text{Fo} = 0$. We introduce the notation

$$\bar{T} (\xi, s) = \int_0^\infty T (\xi, X) \exp (- sX)\, dX .$$

Then for unique passage to the domain of Laplace transforms we must prescribe initial conditions for the one-sided dimensionless coordinate X:

$$[T (\xi, X)]_{X=0} = T_0 , \quad \left(\frac{\partial T}{\partial X}\right)_{X=0} = f_1 (\xi) ,$$

and with the boundary conditions

$$[T(\xi, X)]_{\xi=\pm 1} = \varphi(X) , \quad \left(\frac{\partial T}{\partial \xi}\right)_{\xi=0} = 0 \tag{4}$$

the formulated problem in the case $f_1(\xi = 0)$ is reduced, for $\bar{T}(\xi, s)$, to the form

$$\frac{d}{d\xi}\left(\xi^m \frac{d\bar{T}}{d\xi}\right) - [s\bar{T}(\xi, s) - T_0]\,(w(\xi, m) - \delta s)\,\xi^m + \frac{q_v(\xi)\,R^2}{\lambda}\,\xi^m \bar{f}(s) , \tag{5}$$

$$[\bar{T}(\xi, s)]_{\xi=\pm 1} = \bar{\varphi}(s) , \quad \left(\frac{d\bar{T}}{d\xi}\right)_{\xi=0} = 0 . \tag{6}$$

An approximate solution of Eq. (5) that satisfies boundary conditions (6) exactly is found in the family of the linear manifold

$$\bar{T}_n(\xi, s) = \bar{\varphi}(s) + \sum_{k=1}^{n} \bar{a}_k(s)\,\psi_k(\xi) , \tag{7}$$

where the choice in selecting the basis coordinates of the functional space is restricted only by fulfillment of the homogeneous boundary conditions

$$[\psi_k(\xi)]_{\xi=\pm 1} = 0 , \quad \left(\frac{d\psi_k}{d\xi}\right)_{\xi=0} = 0 . \tag{8}$$

This manifold of Riemannian spaces includes the space of polynomials

$$\psi_k(\xi) = 1 - \xi^{2k} , \quad k = 1, 2, ..., n , \tag{9}$$

where we will find the temperature inside a plane channel ($m = 0$) and a round tube ($m = 1$). We introduce the quantity (7) into the left-hand side of Eq. (5); then the discrepancy will be

$$\varepsilon_n[\bar{a}_1(s), ..., \bar{a}_n(s), \xi] = \frac{d}{d\xi}\left(\xi^m \frac{d\bar{T}_n}{d\xi}\right) - [s\bar{T}_n(\xi, s) - T_0]\,(w(\xi, m) - \delta s)\,\xi^m + \frac{q_v(\xi)\,R^2}{\lambda}\,\xi^m \bar{f}(s) \neq 0$$

and from the requirement $\int_0^1 \varepsilon_n \psi_j d\xi = 0$, $j = 1, 2, ..., n$ we obtain for the coefficients $\bar{a}_1(s)$, ..., $\bar{a}_n(s)$ a determining system that, in matrix form, is equal to

$$(\mathbf{A} + s\mathbf{B} - s^2\delta\mathbf{C})\,\bar{\mathbf{a}}(s) = [T_0 - s\bar{\varphi}(s)]\,(\mathbf{D} - \delta s\mathbf{F}) + \frac{q_v R^2}{\lambda}\,\mathbf{E} , \tag{10}$$

where the matrix elements are calculated from the formulas

$$A_{jk} = -\int_0^1 \frac{d}{d\xi}\left(\xi^m \frac{d\psi_k}{d\xi}\right)\psi_j d\xi = \int_0^1 \frac{d\psi_k}{d\xi}\frac{d\psi_j}{d\xi}\,\xi^m\, d\xi = A_{kj} > 0 ,$$

$$B_{jk} = \int_0^1 \psi_k \psi_j \, w(\xi, m) \, \xi^m \, d\xi = B_{kj} > 0 \, , \quad C_{jk} = \int_0^1 \psi_j \psi_k \, \xi^m \, d\xi = C_{kj} > 0 \, ,$$

$$D_j = \int_0^1 w \, \xi^m \psi_j \, d\xi \, , \quad F_j = \int_0^1 \xi^m \psi_j \, d\xi \, , \quad E_j = \int_0^1 \psi_0(\xi) \, \xi^m \, \psi_j \, d\xi \, , \quad q_v(\xi) = q_v \, \psi_0 \, . \tag{11}$$

By calculating the coefficients B_{jk} and D_j for prescribed profiles of the velocity $w(\xi, m)$ and a fixed parameter m we write system (10) in explicit form. For example, for the laminar isothermal flow of a Newtonian fluid

$$w(\xi, m) = \frac{m+3}{2} (1 - \xi^2) \, , \quad \psi_0(\xi) = 1$$

from the truncated system of first order for $m = 1$ we find

$$\bar{a}_1(s) = \frac{[T_0 - s\bar{\varphi}(s)] \, (1.5s - 2\mathrm{Pe}^2)}{s^2 - 1.5\mathrm{Pe}^2 s - 6\mathrm{Pe}^2} + \frac{3q_v R^2}{2\lambda} \frac{\mathrm{Pe}^2 \bar{f}(s)}{6\mathrm{Pe}^2 + 1.5s\mathrm{Pe}^2 - s^2} \, . \tag{12}$$

For large values of Pe where $\delta = \mathrm{Pe}^{-2} \approx 0$ the dimensionless coordinate along the tube length X becomes a one-sided parabolic variable, and in the limit as $\mathrm{Pe}^2 \to \infty$ from (12) we have

$$\bar{a}_1(s) = \frac{4 \, [T_0 - s\bar{\varphi}(s)]}{3 \, (s+4)} + \frac{q_v R^2}{\lambda} \frac{\bar{f}(s)}{s+4} \, , \tag{13}$$

whence by going to the domain of the inverse transforms in the cases $\varphi(X) = T_w \neq T_0$, $f(X) = 0$ and $\varphi(X) = T_0$, $f(X) = 1$ we easily find $a_1(X)$, and the representation (7) in a first approximation leads to the solutions

$$\Theta_1(\xi, X) = \frac{T(\xi, X) - T_w}{T_0 - T_w} = \frac{4}{3} (1 - \xi^2) \exp(-4X) \, , \tag{14}$$

$$T(\xi, X) = T_0 + \frac{q_v R^2}{4\lambda} (1 - \xi^2) \left\{ 1 - \exp(-4X) \right\} . \tag{15}$$

The temperature field (15) in the flow of a heat-generating liquid exactly satisfies the initial conditions at the inlet and the boundary conditions on the tube wall, while after the interval of a transient regime a parabolic distribution that coincides with the exact solution is established.

For small finite Pe numbers, from the first term of (12) we have

$$\bar{a}_1(s) = \frac{1.5s_1^{(1)} - 2\mathrm{Pe}^2}{2s_1^{(1)} - 1.5\mathrm{Pe}^2} \frac{T_0 - s\bar{\varphi}(s)}{s - s_1^{(1)}} + \frac{1.5\tilde{s}_1^{(1)} - 2\mathrm{Pe}^2}{2\tilde{s}_1^{(1)} - 1.5\mathrm{Pe}^2} \frac{T_0 - s\bar{\varphi}(s)}{s - \tilde{s}_1^{(1)}} \, , \tag{16}$$

where $s_1^{(1)}$ and $\tilde{s}_1^{(1)}$ are the roots of the equation $s^2 - 1.5 \, \mathrm{Pe}^2 s - 6 \, \mathrm{Pe}^2 = 0$, i.e.,

$$s_1^{(1)} = 0.5 \, (1.5\mathrm{Pe}^2 - \sqrt{2.25 \, \mathrm{Pe}^4 + 24\mathrm{Pe}^2}) < 0 \, ,$$

$$\tilde{s}_1^{(1)} = 0.5\,(1.5\mathrm{Pe}^2 + \sqrt{2.25\,\mathrm{Pe}^4 + 24\mathrm{Pe}^2}) > 0\,.$$

Instead of solution (14) for $\varphi(X) = T_w \neq T_0$ we find

$$\Theta_1\,(\xi, X) = \frac{T\,(\xi, X) - T_w}{T_0 - T_w} = (1 - \xi^2)\left[\frac{1.5s_1^{(1)} - 2\mathrm{Pe}^2}{2s_1^{(1)} - 1.5\mathrm{Pe}^2}\exp\,(s_1^{(1)}X) + \frac{1.5\tilde{s}_1^{(1)} - 2\mathrm{Pe}^2}{2\tilde{s}_1^{(1)} - 1.5\mathrm{Pe}^2}\exp\,(\tilde{s}_1^{(1)}X)\right]. \tag{17}$$

For the temperature in the flow of the heat-generating liquid, by analogous computations with the second term of (12) instead of formula (15) we obtain

$$T\,(\xi, X) = T_0 + \frac{q_v R^2}{4\lambda}\,(1 - \xi^2)\left\{1 - \frac{1}{\tilde{s}_1^{(1)} - s_1^{(1)}}\left[\tilde{s}_1^{(1)}\exp\,(s_1^{(1)}X) - s_1^{(1)}\exp\,(\tilde{s}_1^{(1)}X)\right]\right\}, \tag{18}$$

where

$$\tilde{s}_1^{(1)} - s_1^{(1)} = \sqrt{2.25\mathrm{Pe}^4 + 24\mathrm{Pe}^2}\,.$$

These solutions for Pe = 4 are reduced to the expressions

$$\Theta_1\,(\xi, X) = \frac{T - T_w}{T_0 - T_w} = [1.202\exp\,(-\,3.492X) + 0.299\exp\,(27.492X)]\,(1 - \xi^2)\,, \tag{19}$$

$$T\,(\xi, X) = T_0 + \frac{q_v R^2}{4\lambda}\,[1 - 0.816\exp\,(-\,3.492X) - 0.184\exp\,(27.492X)]\,(1 - \xi^2)\,. \tag{20}$$

For a bar flow, $W(\xi, m) = w_0$ = const and in Eq. (1) $w(\xi, m) = 1$. Then in the determining system (10) the matrices are $B = C$ and $D = F$. The elements of the matrix $\bar{\mathbf{a}}(s)$ are determined from the Cramer formula

$$\bar{a}_k\,(s) = \frac{\Delta_k^{(D)}\,(z)\,[T_0 - s\bar{\varphi}\,(s)]\,(1 - \delta s)}{\Delta\,(z)} + \frac{q_v R^2}{\lambda}\,\frac{\Delta_k^{(E)}\,(z)\,\bar{f}\,(s)}{\Delta\,(z)}\,, \tag{21}$$

where $\Delta(z) = |A + zB|$; $z = s - \delta s^2$; $\Delta_k^{(N)}(z) = \sum_{j=1}^{n} N_j \Delta_{jk}(z)$; $\Delta_{jk}(z)$ are the algebraic complements of the determinant $\Delta(z)$. Since the matrices A and B are symmetric and positive, the roots of the equation $\Delta(z) = 0$ will be simple and negative. We denote them by $-z_1^{(n)} < 0$, $-z_2^{(n)} < 0$, ..., $-z_k^{(n)} < 0$ $(z_k^{(n)} > 0)$ in ascending order of absolute values; then, expanding the proper fractions $\Delta_k^{(N)}(z)/\Delta(z)$ in simple poles of the denominator, we obtain the synthesis of the elements

$$\bar{a}_k\,(s) = \sum_{i=1}^{n}\frac{\Delta_k^{(D)}\,(-\,z_i^{(n)})}{\Delta'\,(-\,z_i^{(n)})}\,\frac{[T_0 - s\bar{\varphi}\,(s)]\,(1 - \delta s)}{s - \delta s^2 + z_i^{(n)}} + \frac{q_v R^2}{\lambda}\sum_{i=1}^{n}\frac{\Delta_k^{(E)}\,(-\,z_i^{(n)})}{\Delta'\,(-\,z_i^{(n)})}\,\frac{\bar{f}\,(s)}{s - \delta s^2 + z_i^{(n)}} \tag{22}$$

of the response matrix $\bar{\mathbf{a}}(s)$ in the form of an expansion in sums of blocks, each being additionally reduced to terms of two simple links by transformations of the same type that were carried out to derive formula (16). One link is an ordinary inertial link with a stabilizing response along the tube length while the other is a pseudo-inertial link, and its effect on the solution leads to a rapid rise in the temperature, which does not fit exactly the actual thermophysical process.

Thus, the assumption of one-sidedness of the elliptic coordinate X as in a semi-infinite tube and the additional condition for unique application of the Laplace integral transformation led to a boundary-value problem whose solution in the core of the liquid flow increases with X. Therefore we consider another approach to the solution of the formulated problem where a two-sided elliptic coordinate X is adopted. We prescribe boundary conditions at the beginning $X = 0$ and at the end of the reduced length of the tube $X = L$. An approximate solution of the problem

$$w(\xi, 1)\frac{\partial T}{\partial X} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial T}{\partial \xi}\right) + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2 T}{\partial X^2}\,; \quad w(\xi, 1) = 2(1 - \xi^2)\,, \tag{23}$$

$$[T(\xi, X)]_{\xi=1} = T_{\mathrm{w}}\,, \quad \left(\frac{\partial T}{\partial \xi}\right)_{\xi=0} = 0\,, \quad [T(\xi, X)]_{X=0} = T_0\,, \quad [T(\xi, X)]_{X=L} = T_{\mathrm{w}}\,, \tag{24}$$

that exactly satisfies boundary conditions (24) just in ξ will be sought in the form

$$T_1(\xi, X) = T_{\mathrm{w}} + u(X)\,\psi_1(\xi)\,, \quad \psi_1(\xi) = 1 - \xi^2\,. \tag{25}$$

We introduce the quantity $T_1(\xi, X)$ into Eq. (23) and compose the discrepancy

$$\varepsilon_1[u, u', u'', \xi] = \mathrm{Pe}^{-2}\,u''(X)\,\psi_1(\xi)\,\xi - 2u'(X)(1 - \xi^2)\,\xi\psi_1(\xi) - 4\xi u(X) \neq 0\,.$$

Then from the requirement of orthogonality of the discrepancy to the coordinate axis $\psi_1(\xi)\int_0^1 \varepsilon_1\psi_1 d\xi = 0$ we obtain

$$\frac{d^2u}{dX^2} - 1.5\mathrm{Pe}^2\frac{du}{dX} - 6\mathrm{Pe}^2\,u(X) = 0\,. \tag{26}$$

The general solution of this equation will be

$$u(X) = c_1 \exp(s_1^{(1)}X) + c_2 \exp(\tilde{s}_1^{(1)}X)\,. \tag{27}$$

The integration constants in the solution

$$T_1(\xi, X) = T_{\mathrm{w}} + [c_1 \exp(s_1^{(1)}X) + c_2 \exp(\tilde{s}_1^{(1)}X)](1 - \xi^2)$$

are found by complying with the second part of the boundary conditions (24) at the ends $X = 0$ and $X = L$ for the mass-mean temperature

$$\langle T_1(X)\rangle = \frac{2\int_0^1 T_1(\xi, X)\,w(\xi, 1)\,\xi d\xi}{2\int_0^1 w(\xi, 1)\,\xi d\xi} = 4\int_0^1 T_1(\xi, X)(1 - \xi^2)\,\xi d\xi\,.$$

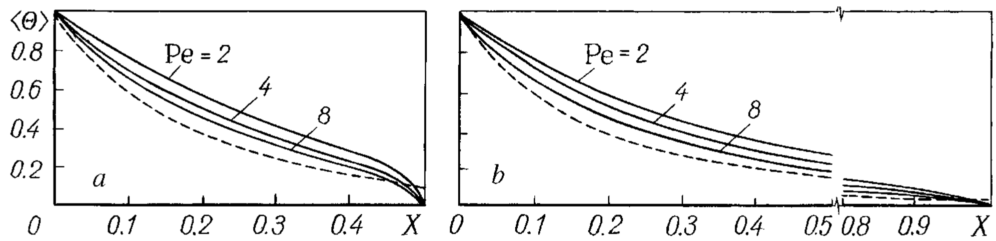

Fig. 1. Change in the mass-mean relative excess temperature with allowance for the heat conduction along the axis of a tube of finite length L = 0.5 (a) and 1.0 (b). The dashed lines show the mass-mean temperature without allowance for the axial heat conduction (Pe $\geq$ 20, $\delta \leq 1/400$).

Then

$$T_1(\xi, X) = T_w + \frac{1.5\,(T_0 - T_w)\,[\exp(\tilde{s}_1^{(1)}X + s_1^{(1)}L) - \exp(s_1^{(1)}X + \tilde{s}_1^{(1)}L)]}{\exp(s_1^{(1)}L) - \exp(\tilde{s}_1^{(1)}L)}\,(1 - \xi^2)\,, \tag{28}$$

$$\langle \Theta_1(X) \rangle = \frac{\langle T_1(X) \rangle - T_w}{T_0 - T_w} = \frac{\exp(s_1^{(1)}L + \tilde{s}_1^{(1)}X) - \exp(\tilde{s}_1^{(1)}L + s_1^{(1)}X)}{\exp(s_1^{(1)}L) - \exp(\tilde{s}_1^{(1)}L)}\,. \tag{29}$$

For the number Pe = 4 the mass-mean relative excess temperature is written by the formula

$$\langle \Theta_1(X) \rangle = \frac{\exp(27.492L - 3.492X) - \exp(27.492X - 3.492L)}{\exp(27.492L) - \exp(-3.492L)}\,. \tag{30}$$

For the numbers Pe = 2 and 10, in the solution (29) we will have, respectively, $s_1^{(1)} = -2.745$, $\tilde{s}_1^{(1)} = 8.745$ and $s_1^{(1)} = -3.899$, $\tilde{s}_1^{(1)} = 153.895$, while for Pe = 20 we find $s_1^{(1)} = -3.974$ and $\tilde{s}_1^{(1)} = 603.974$. With further increase in the Pe number the value of $s_1^{(1)}$ approaches $s_1 = -4$, which coincides with the eigenvalue of Eq. (23) without the term $\mathrm{Pe}^{-2}\partial^2 T/\partial X^2$ when the approximate temperature field is determined by the solving algorithm [3] from a single principal spectrum in the family of the linear manifold (7).

Plots of the change in the mass-mean temperature (29) for the reduced tube lengths L = 0.5 and L = 1.0 and the numbers Pe = 2, 4, and 8 are shown in Fig. 1. The figure gives curves of the exact mass-mean temperature $\langle \Theta(X) \rangle$ from results of calculating the problem inside a semi-infinite tube without allowance for the heat conduction along the flow of the liquid [3].

It follows from Fig. 1a that, inside short tubes with reduced lengths $L \leq 0.5$, the decrease in the temperature in the zone of exit of the liquid depends substantially on the additional boundary conditions at the end of the tube $X = L$. For rather long tubes (see Fig. 1b), this boundary condition becomes natural, and no special thermal loadings are required at their ends, as is adopted in classical formulations of the problems.

Since with increase in the Pe number $\tilde{s}_1^{(1)} > 0$ becomes arbitrarily large and $s_1^{(1)} < 0$ tends to -4, by the transformation of solution (29)

$$\langle \Theta_1(X) \rangle = \frac{\langle T_1(X) \rangle - T_w}{T_0 - T_w} = \frac{\exp(\tilde{s}_1^{(1)}L)\left\{\exp(s_1^{(1)}X) - \exp(s_1^{(1)}L)\exp[\tilde{s}_1^{(1)}(X - L)]\right\}}{\exp(\tilde{s}_1^{(1)}L)\,[1 - \exp(s_1^{(1)}L)\exp(-\tilde{s}_1^{(1)}L)]} \tag{31}$$

for large values of Pe with allowance for $X < L$ we obtain $\exp[\tilde{s}_1^{(1)}(X - L)] \approx 0$ and $\exp(-\tilde{s}_1^{(1)}L) \approx 0$, and the mass-mean temperature is reduced to the approximate expression

$$\langle T_1(X)\rangle = T_w + (T_0 - T_w)\exp(-4X) . \tag{32}$$

As an example for channels with a two-dimensional profile of the cross section that is symmetric to the axis ξ ($\xi = x/h$) we consider the problem of heat transfer in a prismatic channel of the equilateral triangular cross section $D\left\{ y \le \frac{b}{h}x,\ y \le -\frac{b}{h}x,\ 0 \le x \le h,\ \beta = \frac{h^2}{b^2} = 3 \right\}$:

$$w(\xi,\eta)\frac{\partial T}{\partial X} = \frac{\partial^2 T}{\partial \xi^2} + \beta\frac{\partial^2 T}{\partial \eta^2} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2 T}{\partial X^2},\quad X = \frac{1}{\mathrm{Pe}}\frac{z}{h},\quad \mathrm{Pe} = \frac{w_0 h}{a}, \tag{33}$$

$$[T(\xi,\eta,X)]_{X=0} = T_0,\quad [T(\xi,\eta,X)]_{X=L} = T_w\ [T(\xi,\eta,X)]_\Gamma = \varphi(X), \tag{34}$$

where the temperature on the channel walls $\varphi(X)$ changes rather quickly; in any case, it takes on a constant value $\varphi(X) = T_w$ on the interval of the second part of the channel length L. For the equilateral triangular region D, the composite boundary function $\omega(\xi, \eta)$ and the exact expression for the velocity distribution of the isothermal flow of the liquid will be

$$\omega(\xi,\eta) = (\xi^2 - \eta^2)(1-\xi),\quad W(\xi,\eta) = w_0 w = 15 w_0(\xi^2 - \eta^2)(1-\xi),\ \max W = 2.22 w_0 . \tag{35}$$

The representation of the approximate solution in the form

$$T(\xi,\eta,X) = \varphi(X) + u(X)\,\omega(\xi,\eta),\quad -1 \le \eta = y/b \le 1,\quad 0 \le \xi = x/h \le 1, \tag{36}$$

leads, with constant boundary conditions ($\varphi(X) = T_w$), to the expressions

$$T(\xi,\eta,X) = T_w + \frac{10.5(T_0 - T_w)[\exp(\tilde{s}_1 L + s_1 X) - \exp(s_1 L + \tilde{s}_1 X)]}{\exp(\tilde{s}_1 L) - \exp(s_1 L)}(\xi^2 - \eta^2)(1-\xi), \tag{37}$$

$$\langle\Theta(X)\rangle = \frac{\langle T_1(X)\rangle - T_w}{T_0 - T_w} = \frac{\exp(\tilde{s}_1 L + s_1 X) - \exp(s_1 L + \tilde{s}_1 X)}{\exp(\tilde{s}_1 L) - \exp(s_1 L)}, \tag{38}$$

where

$$s_1 = 0.5\left(1.636\mathrm{Pe}^2 - \sqrt{(1.636\mathrm{Pe}^2)^2 + 168\mathrm{Pe}^2}\right) < 0 ,$$

$$\tilde{s}_1 = 0.5\left(1.636\mathrm{Pe}^2 + \sqrt{(1.636\mathrm{Pe}^2)^2 + 168\mathrm{Pe}^2}\right) > 0 .$$

The mass-mean temperature (38) for the number Pe = 4 is written in the form

$$\langle\Theta(X)\rangle = \frac{\langle T(X)\rangle - T_w}{T_0 - T_w} = \frac{\exp(42.128L - 15.952X) - \exp(42.128X - 15.952L)}{\exp(42.128L) - \exp(-15.952L)} . \tag{39}$$

From the solution (37) we find the heat flux on the wall $x = h$ from the formula

$$q(\eta,X) = \left(-\lambda\frac{\partial T}{\partial x}\right)_{x=h} = -\frac{\lambda}{h}\left(\frac{\partial T}{\partial \xi}\right)_{\xi=1},$$

and then the regularity of the heat transfer at all similar points at the three sides is determined in the form

$$q^{*}(\eta, X) = \frac{q(\eta, X)\,h}{\lambda\,(T_0 - T_w)} = 10.5\,(1 - \eta^2)\,\frac{\exp(\tilde{s}_1 L + s_1 X) - \exp(s_1 L + \tilde{s}_1 X)}{\exp(\tilde{s}_1 L) - \exp(s_1 L)}\,. \quad (40)$$

The changes in the heat flux along the perimeter of the cross section and along the length of a semi-infinite channel that are found in terms of the temperature field in the first and the second approximations without allowance for the heat conduction along the channel axis [3] lead, instead of (40), to the formulas

$$q_1^{*}(\eta, X) = 9.167\,(1 - \eta^2)\exp(-\,25.667X)\,, \quad (41)$$

$$q_2^{*}(\eta, X) = 9.526\,(1 - \eta^2)\,[(1 - 1.310\eta^2)\exp(-\,24.084X) - (0.209 + 4.462\eta^2)\exp(-\,196.812X)]\,, \quad (42)$$

where the regularity of the heat transfer is described with a high degree of accuracy using $q_2^{*}(\eta, X)$.

With increase in the Pe number (Pe > 20, for Pe = 20 $s_1 = -24.738$ and $\tilde{s}_1 = 679.138$) the solution (37) virtually coincides with the expression

$$T(\xi, \eta, X) = T_w + 10.5\,(T_0 - T_w)\,(\xi^2 - \eta^2)\,(1 - \xi)\exp(-\,25.667X)\,,$$

where the number 25.667, just as in formula (41), is the coefficient of the rate of exponential stabilization when the solution is found from a single spectrum without allowance for the conduction along the channel axis.

NOTATION

T, temperature in the liquid flow; t, time; T_0 and T_w, temperature at the inlet and on the wall of the channel; Fo and Pe, Fourier and Péclet numbers; s and p, arguments of the Laplace transforms; λ, thermal conductivity of the liquid; a, thermal diffusivity; ν, kinematic viscosity; R, radius of the round tube; $2R$, thickness of the plane channel; w_0, average velocity of the liquid; δ, δ_1, and δ_2, correction parameters; overscribed bar, Laplace transform with respect to the coordinate of the tube length; asterisk on a symbol, Laplace transform with respect to time.

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